

MATH 2 ASSIGNMENT 4

①

1. Consider the section of the curve $y = \frac{2}{3}(x-1)^{3/2}$ between $x=1$ and $x=2$

a) Evaluate the length of this section

b) Evaluate the volume generated by rotating this section through a complete revolution about the x -axis

Soln:

a) Length of the section

$$y = \frac{2}{3}(x-1)^{3/2} \quad x=1 \text{ to } x=2$$

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{2}{3}(x-1)^{3/2} \right]$$

$$= \frac{2}{3} \frac{d}{dx} \left[(x-1)^{3/2} \right]$$

$$= \frac{2}{3} \left[\frac{3(x-1)^{1/2}}{2} \frac{d}{dx} (x-1) \right]$$

$$= \frac{2}{3} \cdot \frac{3(x-1)^{1/2}}{2} \cdot 1$$

$$= (x-1)^{1/2}$$

$$\left[\frac{dy}{dx} \right]^2 = \left[(x-1)^{1/2} \right]^2 = x-1$$

To find the length

We need,

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

here, $a=1$, $b=2$

$$\Rightarrow \int_1^2 \sqrt{1+(x-1)} dx$$

$$\Rightarrow \int_1^2 x dx \Rightarrow \left[\frac{x^{1+1}}{1+1} \right]_1^2 \Rightarrow \left[\frac{x^2}{2} \right]_1^2 = \left[\frac{4}{2} - \frac{1}{2} \right] = \underline{\underline{\frac{3}{2} \text{ units}}}$$

b) Volume of the section

$$V = \pi \int_a^b (y)^2 dx$$

here $a=1$, $b=2$

$$y = \frac{2}{3} (x-1)^{3/2}, \quad y^2 = \left[\frac{2}{3} (x-1)^{3/2} \right]^2 = \frac{2^2 \cdot (x-1)^3}{3^2}$$

$$y^2 = \frac{4 \cdot (x-1)^3}{9}$$

$$V = \pi \int_1^2 \frac{4 \cdot (x-1)^3}{9} dx$$

$$= \pi \cdot \frac{4}{9} \int_1^2 (x-1)^3 dx$$

$$= \pi \cdot \frac{4}{9} \cdot \frac{1}{4}$$

$$= \frac{\pi}{9}$$

$$\underline{\underline{\text{Volume} = \frac{\pi}{9} \text{ Cubic units}}}$$

2. A curve passes through the point $(3, \frac{2}{3} \ln(5) - \frac{3}{4} \ln(15))$ and its slope is given by the functional expression

$$\frac{dy}{dx} = \frac{18-x}{12x^2-7x-12}$$

Determine the equation of the curve

Soln:

$$dy = \frac{18-x}{12x^2-7x-12} dx$$

$$\int dy = \int \frac{18-x}{12x^2-7x-12} dx$$

Taking partial fraction

$$\frac{18-x}{12x^2-7x-12} = \frac{-3}{4x+3} + \frac{2}{3x-4}$$

$$\int dy \Rightarrow \int \left(-\frac{3}{4x+3} + \frac{2}{3x-4} \right) dx$$

$$\int dy \Rightarrow -\int \frac{3}{4x+3} dx + \int \frac{2}{3x-4} dx$$

$$y = -\frac{3}{4} \ln(4x+3) + \frac{2}{3} \ln(3x-4) + c$$

given, curve passes through the point $(3, \frac{2}{3} \ln(5) - \frac{3}{4} \ln(15))$
we get,

$$c = 3$$

$\therefore$ The equation of the curve is,

$$y = -\frac{3}{4} \ln(4x+3) + \frac{2}{3} \ln(3x-4) + 3$$